Physics 371 Spring 2017 Prof. Anlage
Modern Physics Review

JJ Thomson charge-to-mass measurement in E, B fields: $q/m = \frac{v}{RB}$.

Millikan oil droplet experiment: revealed the quantization of electric charge.

Blackbody radiation, Stefan-Boltzmann law: $R_{Total} = \sigma T^4$, $\sigma = 5.6703 \times 10^{-8} \frac{W}{m^2 K^4}$.
Wien displacement law says that $\lambda_{max} T = 2.898 \times 10^{-3} m - K$.
Radiation power per unit area related to the energy density of a blackbody: $R(\lambda) = \frac{c}{4}\rho(\lambda)$.
Rayleigh-Jeans (classical equipartition argument) law $\rho(\lambda) = 8\pi k_B T/\lambda^4$ leads to the 'ultraviolet catastrophe'.
Planck blackbody radiation (treat the atoms as having discrete energy states, and the light as having energy $E = hf$): $\rho(\lambda) = \frac{8\pi hc/\lambda^5}{e^{hc/\lambda k_B T} - 1}$, $h = 6.626 \times 10^{-34} J - s$.

Photoelectric effect and the concept of light as a particle (photon with $E = hf$): $hf = eV_0 + \phi$. Photon collides with one electron and transfers all of its energy, $-V_0$ is the stopping potential.

X-ray production by Bremsstrahlung with cutoff $\lambda_{min} = \frac{1240}{V}\, nm$ (Duane-Hunt Rule), explained by Einstein as inverse photoemission with $\lambda_{min} = \frac{hc}{eV}$. Sharp emission lines arise from quantized energy levels in the 'core shells' of atoms.
Bragg reflection of x-rays from layers of atoms in crystals: $n\lambda = 2d \sin\theta$, where $n = 1, 2, 3, \ldots$, d is the spacing between the parallel layers.

Rutherford scattering (Phys 410) suggested that positive charge is concentrated in a very small volume – the nuclear model of the atom.

Empirical rule for light emission from hydrogen $\frac{1}{\lambda_{mn}} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$, Rydberg constant $R = R_H = 1.096776\ \times 10^7\ \frac{1}{m}$ for Hydrogen.
Bohr model of the hydrogen atom (assumes stationary states, light comes from transitions between stationary states, electron angular momentum in circular orbits is quantized): $\left|\vec{L}\right| = |\vec{r} \times m\vec{v}| = mvr = n\hbar$, with $n = 1, 2, 3, \ldots$, Radius of circular orbits: $r_n = \frac{n^2 a_0}{Z}$ with $a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{me^2} = 0.529$ Å, Total energy of Hydrogen atom: $E_n = -E_0 \frac{Z^2}{n^2}$, with $E_0 = \frac{mc^2 \left(e^2/4\pi\varepsilon_0\right)^2}{2\ (\hbar c)^2} = \frac{mc^2}{2}\alpha^2 = 13.6\ eV$, $\alpha = \frac{e^2/4\pi\varepsilon_0}{\hbar c} \cong \frac{1}{137}$ is called the 'fine structure constant'. Explains the Hydrogen atom emission spectrum but not multi-electron atoms.

Davisson-Germer experiment shows that matter (electrons) diffract from periodic structures (Ni atoms on a surface) like waves. It is clear that matter has a strong wave-like character when measured under appropriate conditions.
deBroglie proposed the wavelength of matter waves as $\lambda_{dB} = h/p$, where p is the linear momentum. Classical physics should be recovered in the short-λ_{dB} limit – the Correspondence Principle

The time-dependent Schrodinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$;
Separation of variables leads to $\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$ (a property of stationary states);
Time-independent Schrodinger equation: $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\,\psi(x) = E\psi(x)$;
The wavefunction $\Psi(x,t)$ is complex in general and cannot be measured. Born interpretation of the wave function in terms of a probability density $P(x,t) = \Psi^*(x,t)\Psi(x,t)$;
Normalization condition: $\int_{-\infty}^{+\infty}|\psi(x)|^2\,dx = 1$.

Particle of mass m in an infinite square well between $x = 0$ and $x = L$: $E_n = \frac{\hbar^2 {k_n}^2}{2m} = n^2\frac{\pi^2\hbar^2}{2mL^2}$ with $n = 1, 2, 3, \ldots$, and $\psi_n(x) = \sqrt{2/L}\,\sin k_n x$.
Finite square well of height V_0, energy eigenvalues are solutions of the transcendental equation: $\tan\left(\frac{\sqrt{2mE}}{\hbar}a\right) = \sqrt{\frac{V_0 - E}{E}}$ (even parity solutions). Always at least one solution!

Harmonic oscillator: $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + \frac{1}{2}m\omega^2x^2\psi(x) = E\psi(x)$, $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$, where $n = 0, 1, 2, 3, \ldots$, Eigenfunctions: $\psi_n(x) = C_n\,e^{-m\omega^2x^2/2\hbar}\,H_n(x)$, involve the Hermite polynomials multiplying a Gaussian in x.
Classical turning points are inflection points in $\psi(x)$.

General wave uncertainty properties: $(\Delta x)\,(\Delta k) \geq 1/2$, $(\Delta t)\,(\Delta\omega) \geq 1/2$.
Quantum uncertainty properties: $(\Delta x)\,(\Delta p) \geq \hbar/2$, $(\Delta t)\,(\Delta E) \geq \hbar/2$.

Expectation values: $\langle x\rangle = \int_{-\infty}^{\infty}\Psi^*(x,t)\,x\,\Psi(x,t)\;dx$, and for any function of position: $\langle f(x)\rangle = \int_{-\infty}^{\infty}\Psi^*(x,t)\,f(x)\,\Psi(x,t)\;dx$

Linear momentum operator: $p_{op} = -i\hbar\,\frac{\partial}{\partial x}$, Hamiltonian operator: $\mathcal{H}_{op} = \frac{{p_{op}}^2}{2m} + V(x)$, the time independent Schrodinger equation written as an operator equation: $\mathcal{H}_{op}\psi(x) = E\,\psi(x)$.

Step potential $V(x) = \begin{cases} 0 & \text{for } x < 0 \\ V_0 & \text{for } x > 0 \end{cases}$ has reflection rate $R = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2$, and transmission rate $T = \frac{4\,k_1\,k_2}{(k_1 + k_2)^2}$, where $k_1 = \sqrt{2mE}/\hbar$ and $k_2 = \sqrt{2m(E - V_0)}/\hbar$.

Tunneling rate through a barrier $T = \left[1 + \frac{\sinh^2(\alpha a)}{4\frac{E}{V_0}\left(1-\frac{E}{V_0}\right)}\right]^{-1} \approx 16\ \frac{E}{V_0}\left(1 - \frac{E}{V_0}\right)e^{-2\alpha a}$, where a is the barrier width, and $\alpha = \sqrt{2m(V_0 - E)}/\hbar$.